Humidity Definitions

1 Relative humidity
Relative humidity is the ratio of two pressures: \( \% \text{RH} = 100 \times \frac{p}{\text{ps}} \) where \( p \) is the actual partial pressure of the water vapor present in the ambient and \( \text{ps} \) the saturation pressure of water at the temperature of the ambient.

Relative humidity sensors are usually calibrated at normal room temperature (above freezing). Consequently, it generally accepted that this type of sensor indicates relative humidity with respect to water at all temperatures (including below freezing).

Ice produces a lower vapor pressure than liquid water. Therefore, when ice is present, saturation occurs at a relative humidity of less than 100 \%. For instance, a humidity reading of 75 \% RH at a temperature of -30°C corresponds to saturation above ice.

2 Dew point / Frost point temperature
The dew point temperature of moist air at the temperature \( T \), pressure \( P_b \) and mixing ratio \( r \) is the temperature to which air must be cooled in order to be saturated with respect to water (liquid).

The frost point temperature of moist air at temperature \( T \), pressure \( P_b \) and mixing ratio \( r \) is the temperature to which air must be cooled in order to be saturated with respect to ice.

Magnus Formula for dew point (over water):
\[
T_d = \frac{243.12 \times \ln \left( \frac{p_w}{611.2} \right)}{17.62 - \ln \left( \frac{p_w}{611.2} \right)}
\]

Frost point (over ice):
\[
T_f = \frac{272.62 \times \ln \left( \frac{p_i}{611.2} \right)}{22.46 - \ln \left( \frac{p_i}{611.2} \right)}
\]

\( p_w \) – vapor pressure over water
\( p_i \) – vapor pressure over ice

3 Wet bulb temperature
The wet bulb temperature of moist air at pressure \( P_b \), temperature \( T \) and mixing ratio \( r \) is the temperature which the air assumes when water is introduced gradually by infinitesimal amounts at the current temperature and evaporated into the air by an adiabatic process at constant pressure until saturation is reached.

\[ p = (\text{ps} @ Tw) - P_b) \times K \times (T - Tw) \]
\[ K = 0.000662 \text{ C°} \]

Partial pressure (\( p \)) can be calculated from a wet bulb (\( Tw \)) and a dry bulb (\( T \)) measurement:

\[ p = (p_{s @ Tw}) - P_b) \times K \times (T - Tw) \]

\[ K = 0.000662 \text{ C°} \]

continued
4 Vapor concentration

The vapor concentration (density of water vapor in a mixture) – or absolute humidity – is defined as the ratio of the mass of water vapor $M_v$ to the volume $V$ occupied by the mixture.

$$D_v = M_v / V$$

expressed in grams/m3 or in grains/cu ft

This can be derived as follows from the equation $PV = nRT$:

a) $M_v = n \times m_w$

- $n$ = number of moles of water vapor present in the volume $V$
- $m_w$ = molecular mass of water

b) $D_v = M_v / V = n \times m_w / V = m_w \times p / RT$, where:

- $m_w = 18.016$ gram
- $p$ = partial pressure of water vapor [Pa]
- $R = 8.31436$ Pa x m3 / °K x mole
- $T$ = temperature of the gas mixture in °K

$$D_v [g / m3] = p / 0.4615 \times T$$

$$D_v [gr / cu ft] = 0.437 \times D_v [g / m3]$$

5 Specific humidity

The specific humidity (also known as mass concentration or moisture content of moist air) is the ratio of the mass $M_v$ of water vapor to the mass $(M_v + M_a)$ of moist air in which the mass of water vapor $M_v$ is contained.

$$Q = M_v / (M_v + M_a)$$

$Q = p m_w / (p m_w + (P_b – p) m_a)$

- $m_w = 18.016$ gram
- $m_a = 28.966$ gram
- $p$ = partial pressure of water vapor [Pa]
- $P_a$ = partial pressure of dry air [Pa]
- $P_b$ = total or barometric pressure [Pa]

$$Q [g / kg] = 1000 p / (1.6078 P_b – 0.6078 p)$$

$$Q [gr / lb] = 7 \times Q [g / kg]$$

6 Enthalpy

The enthalpy (or energy content) of moist air at pressure $P_b$, temperature $t$ (°C) and mixing ratio $r$ (g/kg) is defined by:

$$h [kJ / kg moist] = 1.00464 \times t + 0.001846 \times r \times t + 2.5 \times r$$

Note: by convention, the enthalpy of dry air ($r = 0$) at 0°C is equal to zero. Negative values of enthalpy are possible and indicate that the energy content of the air / vapor mixture is less than the energy content of dry air at 0°C. The value 7.68 is added to make the temperature of 0°F the reference for enthalpy expressed in BTU / lb

continued
7 Mixing ratio by weight

The mixing ratio \( r \) of moist air is the ratio of the mass \( M_v \) of water vapor to the mass \( M_a \) of dry air with which the water vapor is associated:

\[
r = \frac{M_v}{M_a}
\]

\[
M_v = n \times m_w = m_w \times \frac{p \cdot V}{RT}
\]

\[
M_a = n \times m_a = m_a \times \frac{p_a \cdot V}{RT} = m_a \times \frac{(P_b - p)}{RT}
\]

\[
m_w = 18.016 \text{ gram}
\]

\[
m_a = 28.966 \text{ gram}
\]

\( p \) = partial pressure of water vapor [Pa]

\( p_a \) = partial pressure of dry air [Pa]

\( P_b \) = total or barometric pressure [Pa]

\( R \) = 8.31436 Pa \times m^3 / °K \times mole

\( T \) = temperature of the gas mixture in °K

\[
r = \frac{m_w \cdot p}{m_a \cdot (P_b - p)}
\]

\[
r = 621.97 \times \frac{p}{(P_b - p)} \left[ \frac{\text{g}}{\text{kg}} \right]
\]

\[1 \text{ gr (grain)} = 0.0648 \text{ g (gram)}\]